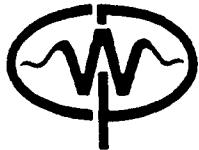


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## Velocity analysis by perturbation

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# Velocity analysis by perturbation

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## ABSTRACT

Prestack depth migration provides a powerful tool for doing velocity analysis in complex media. Both prominent approaches to velocity analysis—depth-focusing analysis and residual-curvature analysis, rely on approximate formulas in order to estimate velocity errors. Generally, these formulas are derived under the assumptions of horizontal reflector, lateral velocity homogeneity, or small offset. Therefore, the conventional methods for updating velocity lack sufficient computational efficiency when velocity has large, lateral variations.

Here, using a perturbation method, we derive an analytic relationship between residual moveout and residual velocity. In our derivation, we impose no limitation on offset, dip, or velocity distribution. Based on this formula, we revise the residual-curvature-analysis method for velocity estimation that involves recursion and iteration. Furthermore, this formula provides the sensitivity and error estimation for migration-based velocity analysis, which is helpful in explaining the reliability of the estimated velocity.

## INTRODUCTION

Prestack depth migration that can handle dipping reflectors and lateral velocity variations is robust in imaging complex structures. In order to process data by this method, one often needs to have a more accurate velocity model than may be obtained from simple velocity-analysis methods, such as normal moveout. Meanwhile, prestack depth migration itself is an attractive tool for doing velocity analysis because of its high sensitivity to the velocity model. Two approaches to migration velocity analysis have been developed: depth-focusing analysis (DFA) and residual-curvature analysis (RCA). Depth-focusing analysis is based on using stacking power to measure velocity error. Residual-curvature analysis is based on residual moveout to measure velocity error.

Migration velocity analysis must address two issues to succeed: (1) how to establish a criterion for knowing if a migration velocity is acceptable; (2) how to update the velocity, if it is unacceptable. In DFA, a migration velocity is acceptable if the difference between migration depth and focusing depth is zero. In RCA, a migration

velocity is acceptable if the difference between imaged depths from different offsets is zero. Quantitatively, these differences can be used to update the velocity. To date, a variety of updating formulas have been developed. These formulas degrade with increasing complexity of media (lateral velocity variation and reflector dip), because of rough approximations used to estimate velocity. Although iteration generally is helpful in obtaining a more accurate velocity, a good approximation not only reduces iteration steps but guarantees the convergence. The conventional formulas for updating velocity were derived under one or more assumptions as follows:

- (i) lateral velocity homogeneity;
- (ii) small offset;
- (iii) horizontal reflector.

In DFA, all three assumptions are used (MacKay and Abma, in 1992). In RCA, the assumptions vary with migration types. For common-shot migration or common-receiver migration, Al-Yahya's formula (1989) used all three assumptions. Lee and Zhang (1992) developed a formula that replaces assumption (iii) by the assumption of small dip. For Common-offset migration, Deregowski's formula (1990) used the first two conditions. [This was proven by Liu and Bleistein (1992).] Both DFA and RCA use assumptions (i) and (ii). Under the assumption of a small offset, the residual moveout can be approximated by a hyperbola or a parabola. However, this approximation is poor when the velocity has an obvious lateral variation.

Conventional approaches, such as DFA and RCA, inspect information on each midpoint individually. This may not be reliable when conflicting events exist because velocity estimation will depend on which event is used. Furthermore, the conventional approaches estimate the stacking velocities that are assumed to equal the rms velocities, and then convert the rms velocities into the interval velocities by Dix' equation or other methods. The stacking velocity is very sensitive to the lateral variation in velocity. When the velocity is laterally variant, the stacking velocity may be very different from the rms velocity.

Because of limitations, conventional approaches to velocity analysis cannot handle complex media (such as the model for Marmousi data). To overcome these limitations, some geophysicists used a macro-model method for estimating the interval velocity directly. A macro-model consists of velocities and velocity interfaces. An initial macro-model is determined from conventional techniques; the macro-model is updated iteratively by using depth-focusing, residual-curvature analysis, or both. The whole process may be accomplished with interpretation based on geological knowledge. The present macro-model method has two significant drawbacks. The first one is that velocity distribution is assumed to be constant in layers. In fact, velocity may vary within a block. The second one is that no efficient formula is used to update velocity. The criterion used now for updating velocity is only qualitative, which results in inaccuracy and inefficiency in velocity estimates.

In this paper, we present a new method for velocity analysis that revises the RCA method. Based on perturbation theory, we derive a quantitative relationship between

residual moveout and error of velocity that is valid for any offset, dip and velocity distribution. A similar formula was proposed by Lafond et al. (1993). However, that formula is restricted to constant-velocity layers. In contrast, our formula can handle any parameterization of velocity. To apply this formula, it is necessary to calculate some quantities that depend on the specular source-receiver pairs for each image point. Here, we use multiple outputs of Kirchhoff migration to determine these quantities efficiently.

Although prestack migration has superior advantages to deal with complex media, its weaknesses should be considered in application. Besides the costly computation, complexity of medium may degrade the effectiveness of the prestack migration method. Here, we discuss the sensitivity of migration-based velocity analysis, and show what factors affect the sensitivity, so that we may estimate the velocity error involved in the velocity analysis. Furthermore, this sensitivity analysis provides an indicator of when *a priori* geological knowledge is necessary for estimating reasonable velocities.

## VELOCITY ESTIMATION

If an incorrect velocity is used in prestack migration, the imaged depths from different offsets at a common-image-gather will differ from each other. In this situation, a residual moveout is observed in migrated data. The principle of velocity analysis is to correct the velocity so that the imaged depths at each common-image-gather are close to for each other. For this purpose, one needs a quantitative relationship between the residual moveout and the velocity error. Here, we use the perturbation method to derive the representation for the residual moveout.

We denote by  $X$  a 2-D vector,  $X = (x, z)$ . Let  $x_s$  be the source position and  $x_r$  be the receiver position on the surface. For any point  $X$  below the surface,  $\tau_s(x_s, X)$  or  $\tau_r(X, x_r)$ , respectively, denote traveltimes from  $x_s$  to  $X$ , or  $X$  to  $x_r$ .

Suppose we know the total reflection travelttime function  $T(y, h)$  (therefore,  $\partial T / \partial y$ ) that depends on midpoint  $y$  and half-offset  $h$ . Given a velocity function  $v(x, z)$ , then, for each  $h$ , the reflector is determined by

$$\tau_s(x_s, X) + \tau_r(X, x_r) = T(y, h), \quad (1)$$

$$\frac{\partial \tau_s}{\partial y} + \frac{\partial \tau_r}{\partial y} = \frac{\partial T}{\partial y}, \quad (2)$$

where  $X \equiv (x, z)$  is the point on the reflector. The reflector is related to offset; for a fixed image location  $x$ , the imaged depth  $z$  can be computed that is a function of  $h$ . If  $v(x, z)$  equals the true velocity, then the imaged depth  $z$  is independent of offset  $h$ ; otherwise—for incorrect velocity— $z$  varies with offset  $h$ . Consequently, the imaged depth provides information on velocity.

Suppose that the velocity distribution  $v$  is characterized by a parameter  $\lambda$ ,

$$v = v(\lambda; X).$$

For example, when  $v(x, z) = v_0 + ax + bz$ ,  $\lambda$  is either  $v_0$ ,  $a$ , or  $b$ .

For a fixed image location  $x$ , we differentiate equation (1) with respect to  $\lambda$ . Noticing that  $y$  and  $z$  are functions of  $\lambda$ , then

$$\left[ \frac{\partial \tau_s}{\partial y} + \frac{\partial \tau_r}{\partial y} \right] \frac{dy}{d\lambda} + \left[ \frac{\partial \tau_s}{\partial \lambda} + \frac{\partial \tau_r}{\partial \lambda} \right] + \left[ \frac{\partial \tau_s}{\partial z} + \frac{\partial \tau_r}{\partial z} \right] \frac{dz}{d\lambda} = \frac{\partial T}{\partial y} \frac{dy}{d\lambda}. \quad (3)$$

By using equation (2), the first term of the left side in equation (2) is balanced by the right-hand term. Therefore,

$$\left[ \frac{\partial \tau_s}{\partial z} + \frac{\partial \tau_r}{\partial z} \right] \frac{dz}{d\lambda} = - \frac{\partial \tau_s}{\partial \lambda} - \frac{\partial \tau_r}{\partial \lambda}.$$

From

$$\frac{\partial \tau_s}{\partial z} = \frac{\cos \theta_s}{v}, \quad \frac{\partial \tau_r}{\partial z} = \frac{\cos \theta_r}{v},$$

where  $\theta_s$  or  $\theta_r$  are angles between the raypath from the source or the receiver, and the vertical at  $X$ , we have that

$$\frac{\cos \theta_s + \cos \theta_r}{v} \frac{dz}{d\lambda} = - \frac{\partial \tau_s}{\partial \lambda} - \frac{\partial \tau_r}{\partial \lambda}.$$

That is,

$$\frac{dz}{d\lambda} = g(x, h), \quad (4)$$

where,

$$g(x, h) = - \left[ \frac{\partial \tau_s}{\partial \lambda} + \frac{\partial \tau_r}{\partial \lambda} \right] \frac{v(\lambda; X)}{\cos \theta_s + \cos \theta_r}. \quad (5)$$

Suppose that the true parameter is  $\lambda^*$  and the true depth is  $z^*$ . If there is a small perturbation  $\Delta\lambda = \lambda^* - \lambda$  between the true parameter and the parameter used in migration, then the imaged depth will have a corresponding perturbation  $\Delta z(x, h) \approx z^* - z(x, h)$ . From equation (4),  $\Delta z$  can be represented by

$$\Delta z = g(x, h) \Delta \lambda. \quad (6)$$

Equation (6) is true for an arbitrary velocity distribution, any dipping reflector, and any offset, which is significantly different from the limited result of conventional RCA.

When the velocity distribution is characterized by multiple parameters,  $\hat{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_n)^T$ , the imaged-depth perturbation will be affected by the perturbations of all these parameters. Therefore, equation (6) is modified by

$$\Delta z(x, h) = \sum_{i=1}^n \frac{\partial z}{\partial \lambda_i} \Delta \lambda_i = \sum_{i=1}^n g_i(x, h) \Delta \lambda_i, \quad (7)$$

where

$$g_i(x, h) = - \left[ \frac{\partial \tau_s}{\partial \lambda_i} + \frac{\partial \tau_r}{\partial \lambda_i} \right] \frac{v(\hat{\lambda}; X)}{\cos \theta_s + \cos \theta_r}. \quad (8)$$

If we could solve for  $\Delta\hat{\lambda}$ , the true parameters can be estimated by

$$\hat{\lambda}^* = \hat{\lambda} + \Delta\hat{\lambda}.$$

Notice that the left side of equation (7) involves the unknown  $z^*$ , so it is necessary to remove  $z^*$  in this equation. The true depth can be approximately replaced by the corrected imaged depth  $z + \Delta z$ ,

$$z^* \approx z(x, h) + \Delta z(x, h) = z(x, h) + \sum_{i=1}^n g_i(x, h) \Delta \lambda_i.$$

Because  $z^*$  is independent of offset, the corrected imaged-depths from different offsets should be close to for each other. Suppose that there are offsets  $h_1, h_2, \dots, h_m$ , and image locations  $x_1, x_2, \dots, x_K$ , then

$$z_j^{(k)} + \Delta z_j^{(k)} = z_j^{(k)} + \sum_{i=1}^n g_{ij}^{(k)} \Delta \lambda_i,$$

where

$$z_j^{(k)} = z(x_k, h_j),$$

$$\Delta z_j^{(k)} = \Delta z(x_k, h_j),$$

$$g_{ij}^{(k)} = g_i(x_k, h_j).$$

We seek  $\Delta \lambda_i$ 's such that the corrected imaged depths have the minimum variance; i.e.,

$$\sum_{k=1}^K \sum_{j=1}^m \left( z_j^{(k)} + \Delta z_j^{(k)} - \bar{z}^{(k)} + \Delta \bar{z}^{(k)} \right)^2 = \min, \quad (9)$$

where

$$\bar{z}^{(k)} = (z_1^{(k)}, z_2^{(k)}, \dots, z_m^{(k)})^T,$$

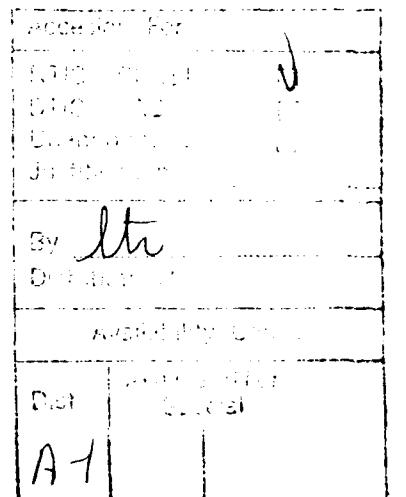
$$\Delta \bar{z}^{(k)} = (\Delta z_1^{(k)}, \Delta z_2^{(k)}, \dots, \Delta z_m^{(k)})^T.$$

Here we use the overline to denote the mean value of a vector over the offset index. For example,

$$\bar{z}^{(k)} = \frac{1}{m} \sum_{j=1}^m z_j^{(k)}.$$

Indroduce the matrix and vector,

$$A^{(k)} \equiv [a_{il}^{(k)}]_{n \times n}, \quad \hat{b}^{(k)} \equiv (b_1^{(k)}, b_2^{(k)}, \dots, b_n^{(k)})^T,$$



where

$$a_{il}^{(k)} = \sum_{j=1}^m \left( g_{ij}^{(k)} - \overline{\hat{g}_i^{(k)}} \right) \left( g_{lj}^{(k)} - \overline{\hat{g}_l^{(k)}} \right),$$

$$b_i^{(k)} = \sum_{j=1}^m \left( g_{ij}^{(k)} - \overline{\hat{g}_i^{(k)}} \right) \left( z_j^{(k)} - \overline{\hat{z}^{(k)}} \right),$$

$$\hat{g}_i^{(k)} = (g_{i1}^{(k)}, g_{i2}^{(k)}, \dots, g_{in}^{(k)})^T;$$

then, as shown in Appendix A, the solution of equation (9) must satisfy the linear equation,

$$\left[ \sum_{k=1}^K A^{(k)} \right] \Delta \hat{\lambda} = - \sum_{k=1}^K \hat{b}^{(k)}. \quad (10)$$

Specifically, if there is only one parameter to be determined, equation (10) will have an explicit solution

$$\Delta \lambda = - \frac{\sum_{k=1}^K \sum_{j=1}^m \left( g_j^{(k)} - \overline{\hat{g}^{(k)}} \right) \left( \hat{z}_j^{(k)} - \overline{\hat{z}^{(k)}} \right)}{\sum_{k=1}^K \sum_{j=1}^m \left( g_j^{(k)} - \overline{\hat{g}^{(k)}} \right)^2}, \quad (11)$$

where

$$g_j^{(k)} = g(x_k, h_j),$$

and

$$\hat{g}^{(k)} = (g_1^{(k)}, g_2^{(k)}, \dots, g_m^{(k)})^T.$$

If the corrected imaged-depths are not close enough to each other, we implement iteration to obtain more accurate parameters. The iteration stops when the variance achieves a given accuracy.

### Calculation of the function $g_i$

The function  $g_i(y, h)$  involves the derivatives of traveltimes with respect to the parameter  $\lambda$ . In the eikonal equation,

$$\left( \frac{\partial \tau}{\partial x} \right)^2 + \left( \frac{\partial \tau}{\partial z} \right)^2 = \frac{1}{v^2(x, z)},$$

if we take the derivative with respect  $\lambda$ , then

$$\frac{\partial \mu}{\partial x} \frac{\partial \tau}{\partial x} + \frac{\partial \mu}{\partial z} \frac{\partial \tau}{\partial z} = \frac{1}{2} \frac{\partial}{\partial \lambda} \left( \frac{1}{v^2(x, z)} \right), \quad (12)$$

where

$$\mu = \frac{\partial \tau}{\partial \lambda}.$$

For each source or receiver,  $\partial \tau / \partial \lambda$  is determined by solving equation (12). Therefore, given an imaged point  $(x, z)$  and a specular source-receiver pair  $x_s$  and  $x_r$ , we can

calculate  $g_i$  from formula (8). However, there is not an explicit formula to represent the specular source-receiver pair from the imaged point for a complex medium. To solve this problem, here, we use the Kirchhoff integral to calculate  $g_i$ . In the Kirchhoff summation, we calculate two inversion outputs which have two different amplitudes. One is the original amplitude; the other is the original one multiplied by the quantity  $g_i$ . Thus, the ratio of the amplitudes of the two outputs will give  $g_i$  at the specular source-receiver position according to the stationary-phase principle, without requiring knowledge of the specular source-receiver pair. This is the same technique as was used to determine the angle of reflection in Kirchhoff inversion. [See Bleistein et al. (1987).]

### Parameterization of Velocity

Although equation (7) holds for any velocity distribution, the solution will be underdetermined if too many unknown parameters are involved. Consequently, it is essential to characterize the velocity distribution by choosing appropriate parameters. Conventionally, one assumes that a velocity model consists of the construction of the macro-model (constant velocities and velocity interfaces). The interfaces divide the whole model into a number of blocks. Here, we replace constant velocity in one block by a linear function that is characterized by three parameters:

$$\lambda_1 + \lambda_2(z - z_0) + \lambda_3(x - x_0),$$

where  $(x_0, z_0)$  is a reference point. Thus, the velocity distribution is written in a form of

$$v(x, z) = v_0(x, z) + \lambda_1 + \lambda_2(z - z_0) + \lambda_3(x - x_0), \quad (13)$$

where  $v_0$  is a background velocity.

An imaged depth only depends on the velocity above it, except for turning rays. Therefore, a recursive algorithm is possible to determine velocity in an individual block. We start from the block nearest surface. In each block, iteration is used to calculate velocity parameters. Given an initial guess for  $\lambda_i$ 's, common-offset depth migration is implemented to obtain imaged depths and  $g_i(x, h)$  in equation (8) for each midpoint and each offset. Equation (10) will give a correction of the parameters. Then by using the updated parameters as initial guess, we correct the velocity again until convergence is achieved. After velocity analysis in one block, we migrate data with the correct velocity, and then pick the block interface from the imaged structure. When we finish determining velocity and velocity interface in one block, we will repeat the same procedure to the next block.

## SENSITIVITY OF MIGRATION VELOCITY ANALYSIS

Velocity analysis by prestack migration uses the difference between the imaged depths from different offsets to correct the velocities, which is represented by equation (10). If the variance defined by (9) is zero, we conclude that the velocity is correct. However, one cannot obtain exactly zero variance. Many factors result in

a non-zero variance: noise in the input data, nonacoustic properties, and inaccurate description of velocity distribution, so on. Even the variance is apparently zero, it is actually not zero because of the errors in picking the imaged depths. The imaged depths in the variance are picked on the migration output, so that the position error in imaged depths is controlled by the resolution of the migration output, which, in turn, depends on wavelength, among other things.

Quantitatively, the matrices in equation (10) that depend on the functions,  $g_i$ , can be used to describe the sensitivity of the velocity error  $\Delta\lambda$  to the variance error. Here, we will derive analytical representations for the simplest cases. For simplicity, we assume that the velocity  $v(x, z)$  consists of a constant background velocity  $v_0$  and a perturbation that is a linear function of depth in one block. Moreover, we assume that the upper boundary of the block is a horizontal line,  $z = d$ ; i.e.,

$$v(x, z) = v_0 + \alpha(\lambda_1 + \lambda_2(z - z_0)), \quad (14)$$

where  $\alpha = 0$  for  $z < d$  and  $\alpha = 1$  for  $z > d$ . The initial guesses are

$$\lambda_1 = \lambda_2 = 0.$$

Suppose that the true parameters are  $\lambda_1^*$  and  $\lambda_2^*$ , and the reflector is a horizontal segment at a depth  $z^*$ . As shown in Appendix B, we obtain the representations for the  $g_i$ 's in equation (8)

$$g_1(x, h) = \frac{h^2 + z^2}{z^2} \frac{z - d}{v_0}, \quad (15)$$

$$g_2(x, h) = \frac{h^2 + z^2}{z^2} \frac{(z - z_0)^2 - (d - z_0)^2}{2v_0}. \quad (16)$$

From the above two equations, we conclude that

$$g_2(x, h) = \frac{(z - z_0)^2 - (d - z_0)^2}{2(z - d)} g_1(x, h).$$

This means that the coefficients in equation (8) are proportional for all image locations and all offsets, so that we cannot solve for  $\lambda_1$  and  $\lambda_2$  separately. Thus, we have a conclusion: *The parameters  $\lambda_1$  and  $\lambda_2$  cannot be determined at the same time if only one reflector segment is used in the velocity-analysis process.*

This conclusion shows that one should avoid solving for  $\lambda_1$  and  $\lambda_2$  at the same time unless well-separated reflector segments are used for velocity analysis.

If the parameter  $\lambda_2$  is given (i.e.,  $\Delta\lambda_2 = 0$ ), then equation (8) is simplified to

$$\Delta z(x, h) = \left( \frac{h^2}{z^2} + 1 \right) \frac{z - d}{v_0} \Delta\lambda_1. \quad (17)$$

When  $\Delta\lambda_1$  is small,  $z(x, h) \approx z^*$ . Therefore, for any different two half-offsets  $h_2$  and  $h_1$  ( $h_2 > h_1$ ), the difference of imaged depths from these two offsets are

$$z(x, h_2) - z(x, h_1) = \left[ \frac{h_2^2}{(z^*)^2} - \frac{h_1^2}{(z^*)^2} \right] \frac{z^* - d}{v_0} \Delta\lambda_1.$$

i.e.,

$$\delta z(x, h_1, h_2) \equiv z(x, h_2) - z(x, h_1) = \frac{(h_2^2 - h_1^2)(z^* - d)}{(z^*)^2 v_0} \Delta \lambda_1. \quad (18)$$

Equation (18) shows that the error in  $\lambda_1$  is proportional to the thickness of the block layer, which is consistent with results in the paper of Liu and Bleistein (1990). When there is a thin layer, the velocity estimation will become unreliable, so that migration-based velocity analysis cannot handle thin layers.

Similarly, if the parameter  $\lambda_1$  is given (i.e.,  $\Delta \lambda_1 = 0$ ), then

$$\delta z(x, h_1, h_2) = \frac{(h_2^2 - h_1^2)(z^* - d)(z^* + d - 2z_0)}{2(z^*)^2 v_0} \Delta \lambda_2. \quad (19)$$

Equation (19) shows that the error in  $\lambda_2$  is determined not only by the thickness of the block layer but by the difference between the central depth of the block layer and the reference depth  $z_0$ .

We have two kinds of the depth errors:  $\Delta z$  and  $\delta z$ . The former is the difference between the true depth and the imaged depth, which reflects migration error due to the velocity error; the latter is the difference between the imaged depths from different offsets, which results in the velocity error in velocity analysis. The ratio of these two depth errors is given by equations (17) and (18),

$$\gamma \equiv \frac{\Delta z(x, h)}{\delta z(x, h_1, h_2)} = \frac{h^2 + (z^*)^2}{(h_2^2 - h_1^2)}. \quad (20)$$

The ratio  $\gamma$  indicates the migration error due to the resolution in migration output. Equation (20) shows that  $\gamma$  increases as the depth increases.

For a general case, we do not have analytical representations for the  $g_i$ 's. But we can calculate the  $g_i$ 's numerically from which to estimate the velocity error. In fact, if there is only one parameter, equation (11) gives an explicit error estimation.

$$|\Delta \lambda| \leq \left[ \frac{\sum_{k=1}^K \sum_{j=1}^m \left( \hat{z}_j^{(k)} - \bar{\hat{z}}^{(k)} \right)^2}{\sum_{k=1}^K \sum_{j=1}^m \left( g_j^{(k)} - \hat{g}^{(k)} \right)^2} \right]^{1/2}, \quad (21)$$

where the Cauchy-Schwarz inequality is used.

## COMPUTER IMPLEMENTATION

We applied our velocity analysis technique to synthetic data. The prestack migration is implemented by Liu's program (1993). The velocity model shown in Figure 1 consists of linear velocity functions and interfaces. Synthetic seismic traces are generated from this model, with 5 offsets ranging from 100 meters to 900 meters. Two of the common offset gathers are shown in Figure 3.

The velocity analysis process is outlined as follows:

In the first layer, the initial guess of velocity is 1500 m/s. With two iterations, the velocity is 2017 m/s. The true value is 2000 m/s.

In the second layer, we assume that velocity has the form  $v(z) = 2017 + \lambda(z - 500)$  m/s, which guarantees a continuous velocity across the first interface. The initial guess is  $\lambda = 0$ . With two iterations, the velocity is

$$v(z) = 2017 + 1.035(z - 500) = 1500 + 1.035z \text{ m/s.}$$

The true velocity is  $v(z) = 1500 + 1.0z$  m/s.

In the left third layer, we assume that velocity has the form  $v(x, z) = 1000 + 1.5z + \lambda x$  m/s. The initial guess is  $\lambda = 0$ . With one iteration, the velocity is  $v(x, z) = 1000 + 1.5z - 0.1x$  m/s. The true velocity is  $v(x, z) = 1000 + 1.5z - 0.2x$  m/s. Despite the error in lateral variation, the stopping criterion is met since the difference of the imaged depths is apparently zero. Therefore, iteration ceases.

In the right third layer, we assume that velocity has the form  $v(z) = 2535 + \lambda(z - 1000)$  m/s. The initial guess is  $\lambda = 1.035$ . With one iteration, the velocity is

$$v(z) = 2535 + 1.55(z - 1000) = 985 + 1.55z \text{ m/s.}$$

The true velocity is  $v(z) = 1000 + 1.5z$  m/s.

In the fourth layer, the velocity is a constant. The initial guess is  $v = 4000$ . With one iteration, the velocity is 3586 m/s. The true velocity is  $v = 3500$  m/s.

After velocity analysis, we obtain the velocity model shown in Figure 2. With this velocity model, we implement migration, and the result is shown in Figure 5 which is close to the migration result with the true velocity, shown in Figure 4.

Also, we test the computation of  $g_1$  in the first layer. The numerical  $g_1$  is computed from the ratio of the amplitudes in the Kirchhoff migration outputs. Then we compare the numerical  $g_1$  with the true value calculated in equation (17). In our test,  $v_0 = 500$  m/s and  $d = 0$ . The result listed in Table 1 shows that our numerical value of  $g_1$  is accurate for all offsets.

Table 1. Test for  $g_1$ . The unit of offset and depth is m/s.

Offset	Imaged depth	Numerical $g_1$	True $g_1$
100	370	0.249	0.251
300	360	0.278	0.282
500	330	0.345	0.346
700	290	0.476	0.475
900	210	0.773	0.783

## CONCLUSION

Velocity analysis by prestack migration can handle complex media for which conventional approaches lack the efficiency in updating velocity. The perturbation method is used here to derive an analytical relationship between the residual moveout and the error in the background velocity. This method is not limited by offset, dip, or velocity distribution. Based on that, we can estimate velocity efficiently. Furthermore, this relationship helps us to estimate the velocity error from the data resolution. This velocity analysis technique also can be generalized to 3D and to converted waves.

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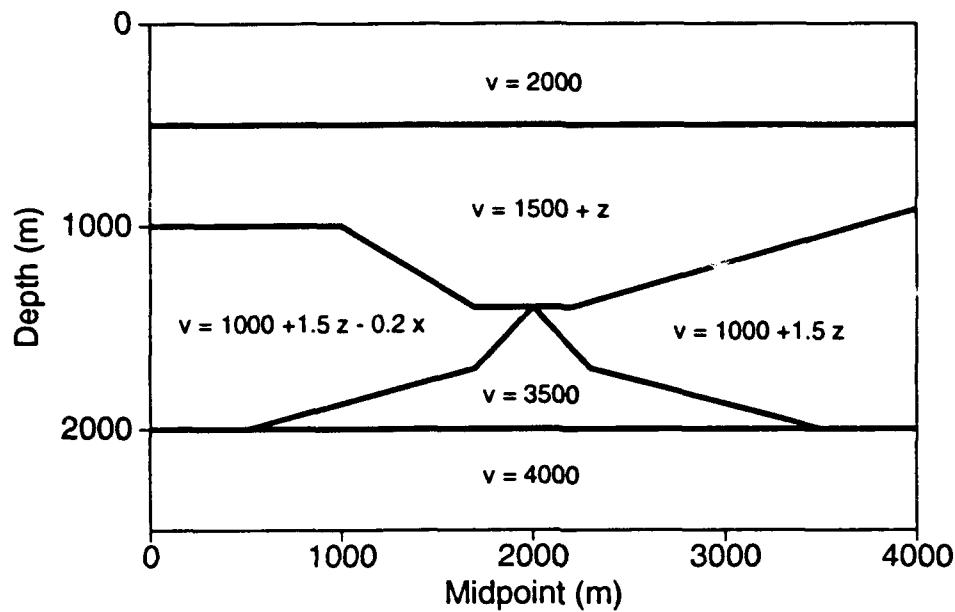


FIG. 1. The true velocity model. The velocity unit is m/s.

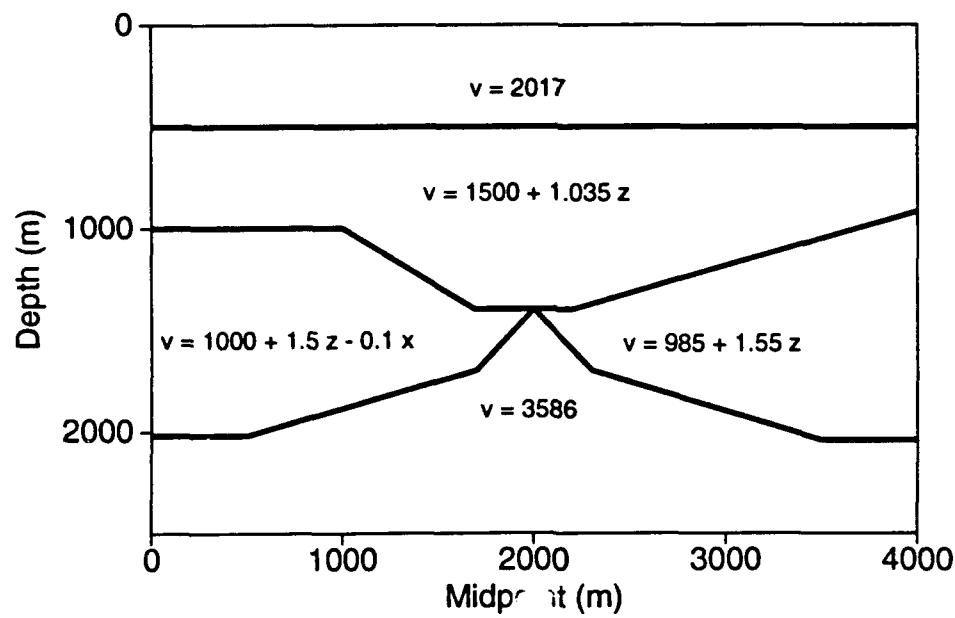


FIG. 2. The velocity model from velocity analysis. The velocity unit is m/s.

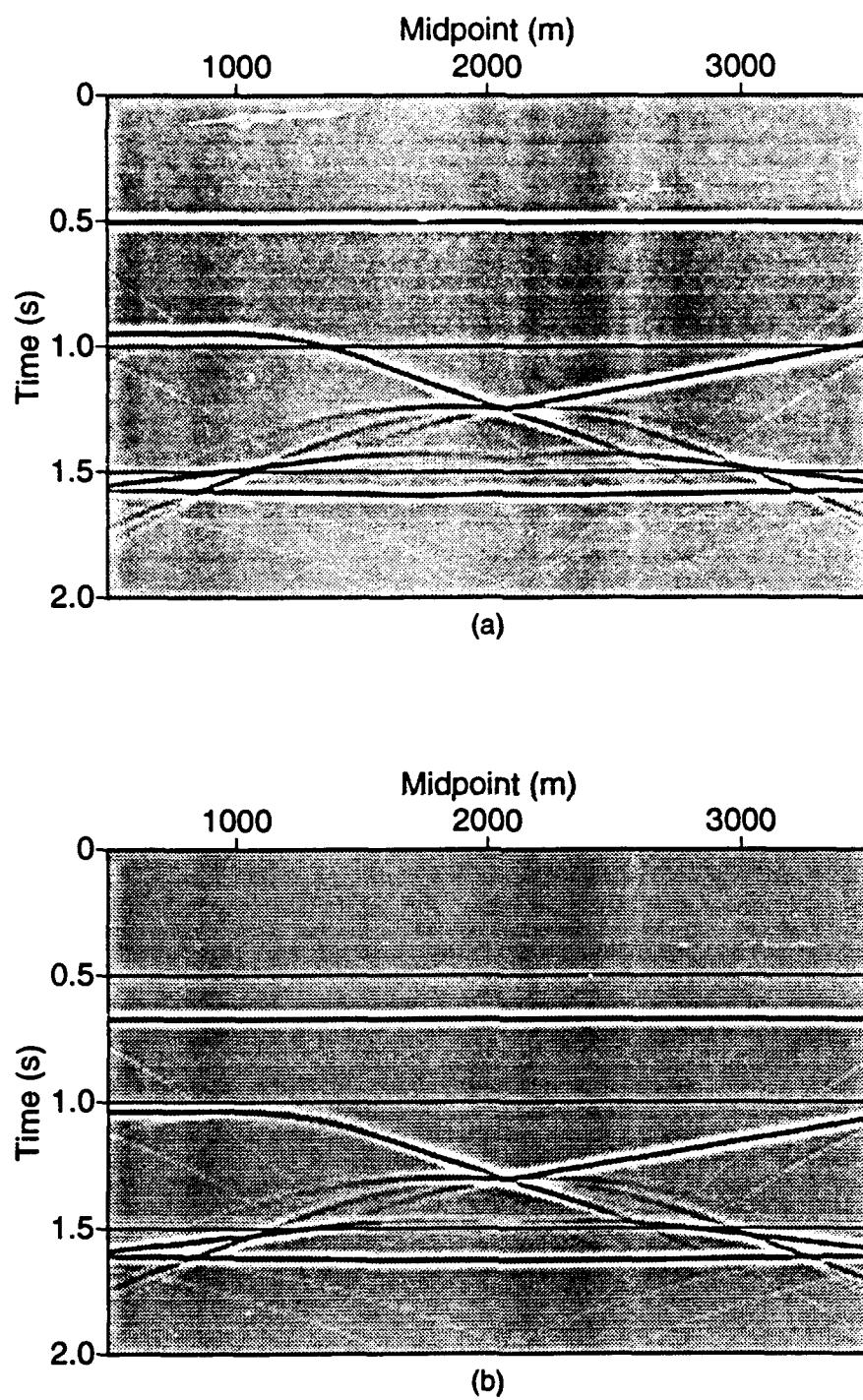


FIG. 3. Synthetic data: (a) with offset of 100 meters and (b) with offset of 900 meters.

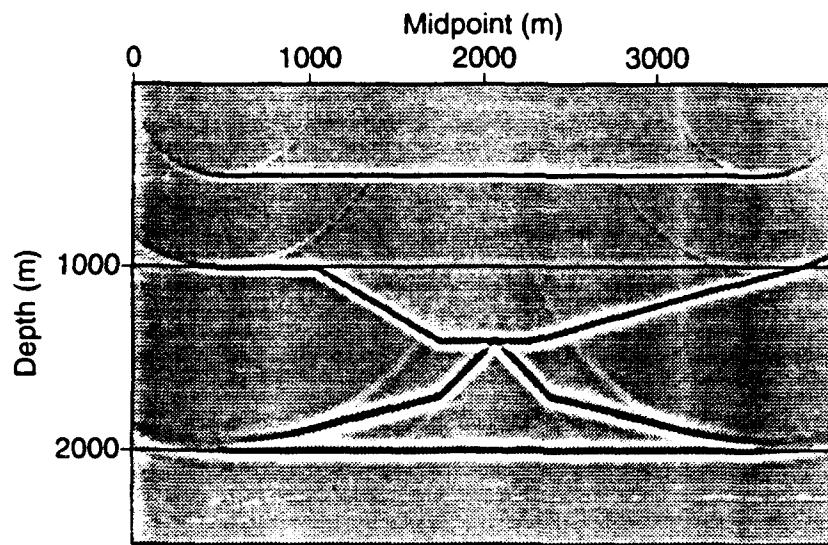


FIG. 4. Prestack migration with the true velocity model in Figure 1. The offset is 100 meters.

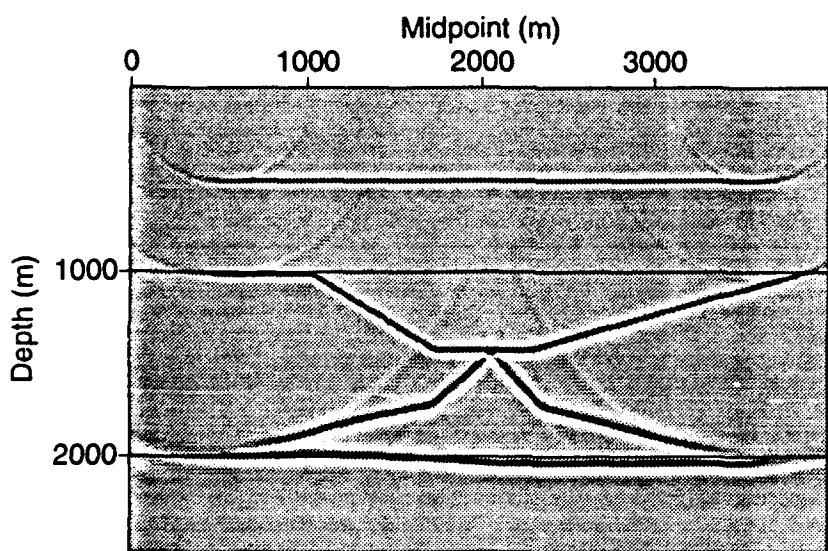


FIG. 5. Prestack migration with the velocity model in Figure 2. The offset is 100 meters.

## DERIVATION OF EQUATIONS (10)

By using the matrix and vector notations on page 5 and 6, we have

$$\begin{aligned}
 \overline{\Delta \hat{z}^{(k)}} &= \frac{1}{m} \sum_{j=1}^m \Delta z_j^{(k)} \\
 &= \frac{1}{m} \sum_{j=1}^m \sum_{i=1}^n g_{ij}^{(k)} \Delta \lambda_i \\
 &= \sum_{i=1}^n \left( \Delta \lambda_i \frac{1}{m} \sum_{j=1}^m g_{ij}^{(k)} \right) \\
 &= \sum_{i=1}^n \left( \Delta \lambda_i \overline{\hat{g}_i^{(k)}} \right), \tag{22}
 \end{aligned}$$

and

$$\begin{aligned}
 z_j^{(k)} + \Delta z_j^{(k)} - \overline{\hat{z}^{(k)} + \Delta \hat{z}^{(k)}} &= z_j^{(k)} - \overline{\hat{z}^{(k)}} + \Delta z_j^{(k)} - \overline{\Delta \hat{z}^{(k)}} \\
 &= z_j^{(k)} - \overline{\hat{z}^{(k)}} + \sum_{i=1}^n \Delta \lambda_i \left( g_{ij}^{(k)} - \overline{\hat{g}_i^{(k)}} \right). \tag{23}
 \end{aligned}$$

Define a function

$$f(\Delta \hat{\lambda}) \equiv \sum_{k=1}^K \sum_{j=1}^m \left( z_j^{(k)} + \Delta z_j^{(k)} - \overline{\hat{z}^{(k)} + \Delta \hat{z}^{(k)}} \right)^2.$$

Thus, finding the minimum of  $f(\Delta \hat{\lambda})$  is equivalent to that its gradient with respect to  $\Delta \hat{\lambda}$  equal 0, i.e.,

$$\frac{\partial f(\Delta \hat{\lambda})}{\partial \Delta \lambda_l} = 0 \quad l = 1, 2, \dots, n. \tag{24}$$

By using equation (23),

$$f(\Delta \hat{\lambda}) = \sum_{k=1}^K \sum_{j=1}^m \left[ z_j^{(k)} - \overline{\hat{z}^{(k)}} + \sum_{i=1}^n \Delta \lambda_i \left( g_{ij}^{(k)} - \overline{\hat{g}_i^{(k)}} \right) \right]^2.$$

So,

$$\begin{aligned}
 \frac{\partial f(\Delta \hat{\lambda})}{\partial \Delta \lambda_l} &= 2 \sum_{k=1}^K \sum_{j=1}^m \left[ z_j^{(k)} - \overline{\hat{z}^{(k)}} + \sum_{i=1}^n \Delta \lambda_i \left( g_{ij}^{(k)} - \overline{\hat{g}_i^{(k)}} \right) \right] \left( g_{lj}^{(k)} - \overline{\hat{g}_l^{(k)}} \right) \\
 &= 2 \sum_{k=1}^K \sum_{j=1}^m \left( z_j^{(k)} - \overline{\hat{z}^{(k)}} \right) \left( g_{lj}^{(k)} - \overline{\hat{g}_l^{(k)}} \right) \\
 &\quad + 2 \sum_{k=1}^K \sum_{j=1}^m \sum_{i=1}^n \Delta \lambda_i \left( g_{ij}^{(k)} - \overline{\hat{g}_i^{(k)}} \right) \left( g_{lj}^{(k)} - \overline{\hat{g}_l^{(k)}} \right). \tag{25}
 \end{aligned}$$

The conditions (24) imply that

$$\sum_{k=1}^K \sum_{j=1}^m \sum_{i=1}^n \Delta \lambda_i \left( g_{ij}^{(k)} - \overline{\hat{g}_i^{(k)}} \right) \left( g_{lj}^{(k)} - \overline{\hat{g}_l^{(k)}} \right) = - \sum_{k=1}^K \sum_{j=1}^m \left( z_j^{(k)} - \overline{\hat{z}^{(k)}} \right) \left( g_{lj}^{(k)} - \overline{\hat{g}_l^{(k)}} \right);$$

i.e.,

$$\left[ \sum_{k=1}^K A^{(k)} \right] \Delta \hat{\lambda} = - \sum_{k=1}^K \hat{b}^{(k)}. \quad (26)$$

This completes the verification that the solution of equation (10) provides the minimum of the left side in (9).

### DERIVATION OF EQUATIONS (14) AND (15)

The solution of equation (12) can be represented by

$$\frac{\partial \tau}{\partial \lambda} = \int_L \frac{\partial}{\partial \lambda} \left( \frac{1}{v(x, z)} \right) dl = - \int_L v^{-2}(x, z) \frac{\partial v(x, z)}{\partial \lambda} dl, \quad (27)$$

where  $L$  is the raypath. Here, to simplify notations, we suppose that all derivatives with respect to  $\lambda$  are evaluated at  $\lambda_1 = \lambda_2 = 0$ . When the background velocity is a constant  $v_0$  as in (13), equation (27) becomes

$$\frac{\partial \tau}{\partial \lambda} = - \frac{1}{\cos \theta} \int_0^z v_0^{-2} \frac{\partial v(x, z')}{\partial \lambda} dz', \quad (28)$$

where  $\theta$  is the angle between the raypath and the vertical. For the velocity function in equation (14), if  $z > d$ , then

$$\frac{\partial v(x, z')}{\partial \lambda_1} = 1,$$

and

$$\frac{\partial v(x, z')}{\partial \lambda_2} = z' - z_0;$$

if  $z < d$ , then

$$\frac{\partial v(x, z')}{\partial \lambda_1} = \frac{\partial v(x, z')}{\partial \lambda_2} = 0.$$

Therefore,

$$\frac{\partial \tau}{\partial \lambda_1} = - \frac{1}{\cos \theta} \int_d^z \frac{1}{v_0^2} dz' = - \frac{1}{\cos \theta} \frac{z - d}{v_0^2}, \quad (29)$$

and

$$\frac{\partial \tau}{\partial \lambda_2} = - \frac{1}{\cos \theta} \int_d^z \frac{z' - z_0}{v_0^2} dz' = - \frac{1}{\cos \theta} \frac{(z - z_0)^2 - (d - z_0)^2}{2v_0^2}. \quad (30)$$

Because the reflector is horizontal, the raypaths from the source and the receiver are symmetric. We have

$$\cos \theta_s = \cos \theta_r = \frac{z}{\sqrt{z^2 + h^2}},$$

Substituting the above formula and equations (29) and (30) into equation (8), we obtain

$$g_1(x, h) = \frac{h^2 + z^2}{z^2} \frac{z - d}{v_0},$$

and

$$g_2(x, h) = \frac{h^2 + z^2}{z^2} \frac{(z - z_0)^2 - (d - z_0)^2}{2v_0}.$$

This completes the verifications of equations (15) and (16).

*Supplementary* ~~Information~~ ~~Form~~

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